IDENTIFICATION AND CONTROL FOR A MODEL HELICOPTER'S YAW

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Abstract— This work presents the design of a yaw controller for a small-scale helicopter. A nominal model identification for the yaw dynamics is performed. Such model includes modeling the effects that cause the variation of torque generated by the engine to the main rotor, which is the principal disturbance source to the yaw control. The identified model is used in the design of two control strategies: PID and LQG. For the disturbance rejection, integral action and a feedforward gain of the disturbance were implemented. Experimental evaluations were conducted to verify the controllers performance under tracking reference and disturbance rejection.

Keywords— Yaw Control, Model Helicopters, System Identification, Disturbance Rejection

Resumo— Este trabalho trata do projeto de um controlador para o movimento de guinada de um modelo reduzido de helicóptero. A identificação do modelo nominal utilizada para a dinâmica de guinada é apresentada, incluindo os efeitos que causam a variação do torque gerado pelo motor, a principal fonte de pertubação para o controle da guinada. O modelo identificado é utilizado no projeto de controladores dos tipos PID e ótimo. Para rejeição de pertubação, uma ação integral e um ganho *feedforward* foram implementados. Por fim, resultados da avaliação experimental mostram o desempenho dos controladores no acompanhamento de sinais referência e na rejeição a perturbações.

Palavras-chave— Controle de guinada, helimodelos, identificação de sistemas, rejeição de pertubações.

1 Introduction

Among several aircrafts applied in aerial robotics projects, the small scale helicopter models, presents several advantages compared to airplanes. The most notable are their (i) agility, (ii) possibility to move with six degrees-of-freedom, (iii) hover ability and (iv) capability of vertical take off and landing, discarding the need of long take-off tracks. Some of the disadvantages of helicopter models are the smaller time of flight autonomy and their inherent instability. Their mathematical models also exhibit several nonlinearities and highly coupled states.

The yaw refers to the rotational movement of the aircraft around the Z axis of the reference coordinate system at the center of gravity, as shown in Figure 1. The yaw controllers actuate in the tail rotor which compensates for the torque produced by the main rotor, influencing in the aircraft stability. The scale effects provides the model helicopters a greater thrust/inertia relation (Gavrilets et al., 2002; Johnson, 1980), making the yaw dynamics more instable. In fact, in the attitude robust control results shown in Bendotti and Morris (1995), the yaw responses oscillate more than the other attitude movements.

Based on the helicopter operation principles, one approach in the design of flight control systems, which has been widely employed, is to consider the system split in some subsystems and the application of several and controllers operating in cascade (Fujiwara et al., 2004; Puntunan and Pamichkun, 2004). Among the advantages of this architecture, its simple structure, low computing load imposed on the flight control system and the possibility to easily perform adjustments in the controllers, not rarely while the aircraft is flying, may be highlighted. However, as the system presents highly coupled states, undesirable parasite responses as the helicopter moves away from a design trim point can occur. In this context, it is important to identify the main disturbance sources for each subsystem of the control structure and use controllers capable of reject these disturbances.

Once the main objective of the yaw controller is to compensate the main rotor torque, the main disturbances for the yaw are clearly the variations on the main rotor torque. In the main rotor control, two approaches are usually considerate: the first one considers the maintenance of the engine throttle constant, which means a constant supplied torque. In the second approach, which was followed in this work, a regulator is designed to keep the main rotor speed constant by varying the throttle. In Martins et al. (2006) it is shown that applying this methodology makes the system more decoupled. Unfortunately, Johnson (1980) reports that assuming the main rotor speed constant, the thrust changes by the main rotor collective pitch in response to vertical motion, generates disturbances to yaw due to the torque variation. Whereas, if the rotor speed is not fixed, the thrust changes would be absorbed by a rotor speed perturbation instead of disturbing the helicopter's yaw motion.

Under these circumstances, this work presents the design of PID and LQG controllers to the helicopter model yaw movement under presence of collective pitch perturbation, as well as the methodology employed and the mathematical model identification. To improve the robustness against disturbances, an integral action based in state augmentation and other based in Mete and Gündes (2004) with anti-windup action are shown. A feedforward term of disturbance value is also verified. A test bench is used to restrict the helicopter movement in the other possible degrees-of-freedom and also to provide a safe condition for the realization of the tests.

This paper is organized as follows: the section 2 describes the test bench, adapted to the model helicopter Raptor 30. The identification and control algorithms are presented in the sections 3 and 4, in this order. At last, the experimental results and conclusions are presented in sections 5 and 6.

2 The Experimental Setup

The model aircraft consists of a Raptor-30 V2 commercially available RC helicopter of 1150 mm total length and approximately 3 kg weight. The 1245 mm main rotor is powered by a two-stroke methanol engine of $6,39$ cm^3. The helicopter itself is a six degrees of freedom system actuated by five Futaba® position servos. Each attitude movement has its respective actuator: the longitudinal cyclic for the pitch (θ), the lateral cyclic for the roll (ϕ), and the tail rotor collective for yaw (ψ). In addition there are two inputs controlling the main rotor thrust: engine throttle for regulating the rotation speed of the main rotor and main rotor collective pitch. The servos are driven by 50 Hz PWM signals whose pulse width varies from 1 ms to 2 ms.

The helicopter is fitted on a three degree of freedom joint, see Figure 1. The mechanical design of the joint allows each of the attitude angles to be made immobilized individually. This is a useful configuration that also allows attitude control experiments with progressive complexity. As the present work deals with the design of a yaw controller, the both of roll and pitch joints were held tight during identification and control, while the yaw joint was limited for 180 degrees of excursion. Besides, the yaw axis of helicopter, that crosses its center of gravity, can be aligned with the correspondent rotation axis of the joint. This prevents a considerable change at yaw dynamics when the helicopter goes to fly untethered. The attachment of the helicopter also provides a

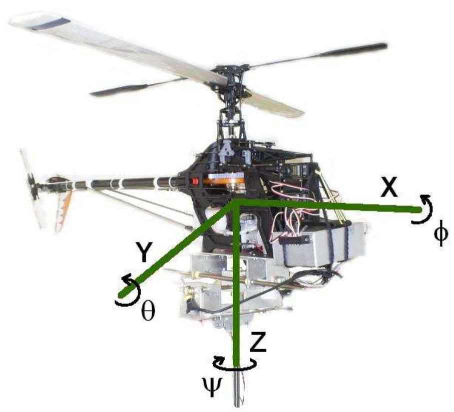


Figure 1: The experimental platform: the helicopter model attached to the 3 degrees-of-freedom base.

safe environment for performing experiments in the laboratory, discarding the need of embedding the full hardware on the model helicopter. The measurements of the joint angles can also be performed using cheap sensors and the control outputs can be computed in a PC.

As shown in the block diagram of Fig. 2, helicopter measurements are provided by two sensory systems. Angular displacements in the joint are provided by potentiometers. Rotor speed is computed from readings of an optical sensor mounted near the tail rotor. Sensor data is transmitted through an RS-485 interface to the control software, running in a Linux-RTAI environment on an IBM-PC machine. The system sampling period is 20 ms. The control software performs digital filtering on the data collected and calculates control output. A simple low-pass filter of first order is utilized for rotor speed measurements and the attitude angles are filtered by a kalman filter that also estimates the respective angular velocities. The output signal is sent to a microcontroller (μC), which is responsible for driving the servos. The system operates in a master-slave architecture, where the IBM-PC always performs the role of master and the microcontrollers are slaves. Communication between both parts exhibits no deadlocks and is fault-tolerant.

3 Identification Setup

Once helicopters are inherently unstable systems, a PID controller in cascade architecture was tunned online to perform the identification tests. The architecture consists of a PI controller for the yaw velocity in the inner-loop and a PD controller at the outer-loop for the yaw position as show the Figure 3. The easy real time controller tuning and the improved disturbance rejection of this structure was useful once the system performance could be evaluated as the main rotor speed rose to its

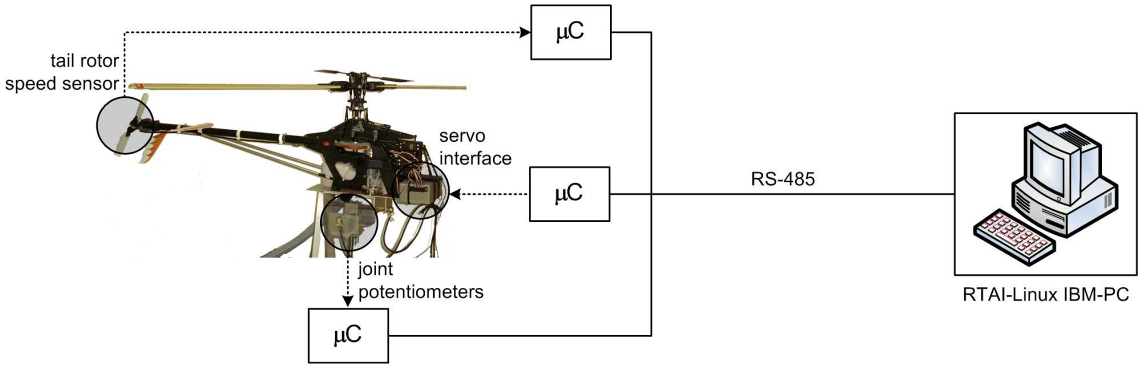


Figure 2: Block diagram of the experimental setup.

nominal value.

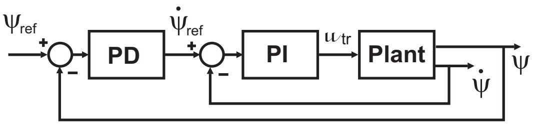


Figure 3: Cascade controller structure.

In order to design advanced controllers with satisfactory performance, a dynamic model that describes the yaw as a function of tail rotor collective u_{tr} and collective pitch u_{mr} must be available. As the principal intent of a yaw controller is to compensate the main rotor torque by maintaining the aircraft yaw stationary, a model for angular yaw velocity $\dot{\psi}$ is more representative. So, the dynamical model in its more general form is given by a state-space model of the form:

$$\mathbf{x}(k+1) = \mathbf{f}\left(\mathbf{x}(k), u_{tr}(k), u_{mr}(k)\right),$$
$$\dot{\psi}(k) = h\left(\mathbf{x}(k)\right).$$

Note that at this point, the state variable x and its dynamical equation f need to be identified. As the general case is hard to solve, it is assumed that the rotor operates around a nominal condition, as hovering, modeled by

$$\dot{\psi} = \bar{\dot{\psi}} + \delta_{\dot{\psi}}, \qquad (1)$$
$$u_{tr} = \bar{u}_{tr} + \delta_{u_{tr}}, \quad u_{mr} = \bar{u}_{mr} + \delta_{u_{mr}},$$

where $\bar{\dot{\psi}}$, $\bar{u}_{tr}$ and $\bar{u}_{mr}$ are the nominal operating points and $\delta_{\dot{\psi}}$, $\delta_{u_{tr}}$ and $\delta_{u_{mr}}$ are the variations. Hence, the following discrete model around nominal operation point may be employed:

$$A(q)\delta_{\dot{\psi}}(k) = B_{tr}(q)\delta_{u_{tr}}(k) + B_{mr}(q)\delta_{u_{mr}}(k). \quad (2)$$

An assumption made in the model described by (2) is that servos dynamics are negligible, because they are much faster than the helicopter dynamics. If there is any low frequency influence in the rotor speed, that effect would be modeled in the identification procedure.

The identification experiment was performed by varying the yaw angle references implying in changes in the tail rotor collective actuated by the inner loop velocity controller. As it's desired a good performance even under collective pitch disturbances, it was excited randomly under the hover zone of operation. To do that, the model was taken into hover nominal conditions, which, as the manufacturer recommends, main rotor speed $\bar{\Omega} = 1510rpm$ and collective pitch around 6 *degrees* which means $\bar{u}_{mr} = 50$. Done that, the identification procedure starts properly making the yaw angle reference and the collective pitch vary over 160 seconds. Figure 4 (a) shows the position references and (b) both collective pitch and tail rotor collective inputs. Among the data collected, the values from the first 80 seconds were used in the identification procedure and the further were reserved for model validation.

The linear model was computed using the subspace method available in MATLAB® System Identification Toolbox. The operation point values applied correspond to a $\bar{\dot{\psi}} = 0\ deg/s$, $\bar{u}_{mr} = 50$. $\bar{u}_{tr}$ is assumed as the mean value over the identification experiment. The Figure 4 (c) shows the data from the identification procedure for a second order model, which exhibited satisfactory results. Little improvement was observed for higher order models.

4 Control Design

Although the model identified represents the yaw velocity dynamics, it's desired to control the yaw angle too. This can be done by the use of a cascade structure like as the presented in Figure 3 or by extending the system to obtain a model of yaw position, which requires just one controller.

The identified system described by (2) can be rewritten in state-space form:

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{G_{tr}}\delta_{u_{tr}}(k) + \mathbf{G_{mr}}\delta_{u_{mr}}(k),$$
$$\delta_{\dot{\psi}}(k) = \mathbf{C}\mathbf{x}(k), \qquad (3)$$

where the vector state $\mathbf{x}$ refers to the linearized model and has the same order of $A(q)$ in (2).

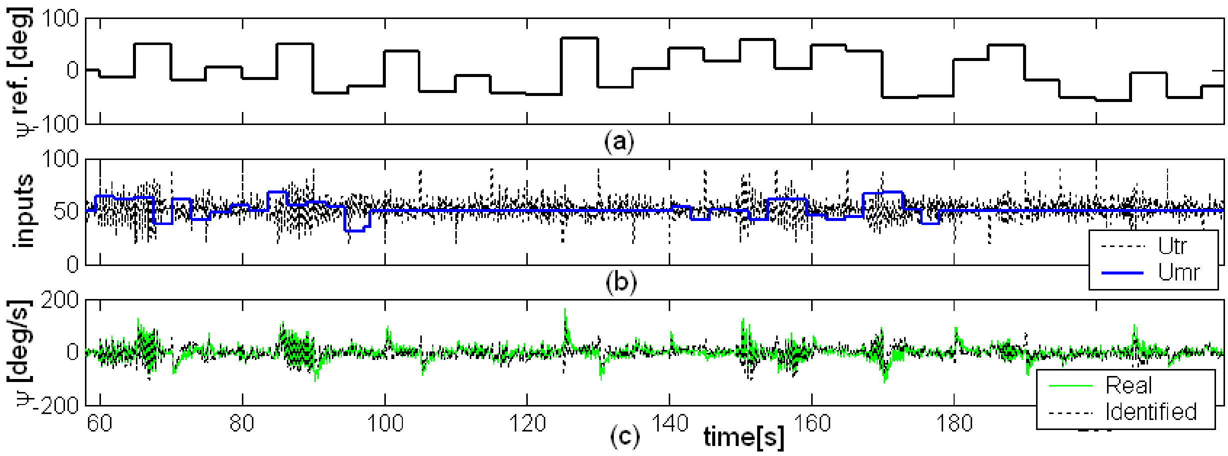


Figure 4: Experimental data and identification results.

If a state for small-signal yaw position δ_ψ is added, the system can be described with the new state variable $\mathbf{z}(k) = [\mathbf{x}^T(k)\ \delta_\psi]^T$:

$$\mathbf{z}(k+1) = \underbrace{\begin{bmatrix} \mathbf{F} & \mathbf{0}_{2\times1} \\ T_s.\mathbf{C} & 1 \end{bmatrix}}_{\mathbf{F}'} \mathbf{z}(k) + \begin{bmatrix} \mathbf{G_{tr}} \\ 0 \end{bmatrix} \delta_{u_{tr}} \quad (4)$$
$$+ \begin{bmatrix} \mathbf{G_{mr}} \\ 0 \end{bmatrix} \delta_{u_{mr}}$$
$$\delta_\psi(k) = \mathbf{\Phi z}(k),$$

where T_s is the sample period and $\mathbf{\Phi} = [\mathbf{0}_{2\times1}\ 1]$.

The controllers are designed based on this small-signal model, which represents the yaw dynamics around the operating point. The controller's inputs and outputs are treated according to (1).

4.1 Linear Quadratic Control

In the present case, given the system described by (4), the regulator that minimizes the cost function

$$V(\mathbf{z}, \delta_{u_{tr}}, k) = \sum_{i=k}^{\infty} \mathbf{z}(i)^T \mathbf{Qz}(i) + \delta_{u_{tr}}(i)^T \mathbf{R} \delta_{u_{tr}}(i)$$

is known in the literature as steady-state linear quadratic regulator. This regulator is obtained by state feedback, whose gains are computed by solving the respective algebraic Riccati equation (Dorato et al., 1995). In the special case of this work, $\mathbf{z}$ is estimated by a Kalman filter. The result is a linear quadratic gaussian regulator (LQG).

However, the linear quadratic regulator goal is to drive all states to zero. If there is a reference input $r(k)$, it can be achieved by adding a gain $\bar{N}$ multiplying the reference input that drives the system to a desired output (Figure 5). This approach is widely employed and detailed in Franklin et al. (1998). The control law is in the form:

$$u(k) = -\mathbf{Kx}(k) + \bar{N}r(k). \quad (5)$$

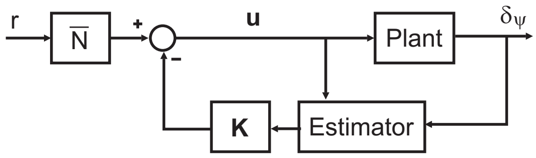


Figure 5: Block diagrams for the reference input structure

4.2 Integral Action

Uncertainties associated with the model and the presence of a constant disturbance $\delta_{u_{mr}}$ make the steady-state output error becomes not null. In order to address this problem, an approach consists in including an integral action in the optimal controller. Usually, a standard integral-action is designed based on augmenting the plant to include the integrator's states in state-feedback.

Therefore, the integral action of the error output can be computed using the augmented state

$$\mathbf{v}(k) = \begin{bmatrix} \mathbf{z}(k+1) - \mathbf{z}(k) \\ \mathbf{\Phi z}(k) \end{bmatrix},$$

which produces the following state-space model

$$\mathbf{v}(k+1) = \begin{bmatrix} \mathbf{F}' & \mathbf{0}_{3\times1} \\ \mathbf{\Phi} & 1 \end{bmatrix} \mathbf{v}(k) \quad (6)$$
$$+ \begin{bmatrix} \mathbf{G_{tr}} \\ 0 \end{bmatrix} \underbrace{(\delta'_{u_{tr}}(k+1) - \delta'_{u_{tr}}(k))}_{\eta'_{u_{tr}}(k)}$$
$$+ \begin{bmatrix} \mathbf{G_{mr}} \\ 0 \end{bmatrix} \delta'_{u_{mr}}(k+1) + \begin{bmatrix} \mathbf{0} \\ -1 \end{bmatrix} r(k).$$

Hence, for that augmented system, the minimization of $V(\mathbf{v}, \eta_{u_{tr}}, k)$ is obtained by the following control law

$$\delta_{u_{tr}}(k) = -\mathbf{K}_1\mathbf{z}(k) - \frac{K_2}{q-1}(\mathbf{\Phi z}(k) - r(k)), \quad (7)$$

where $\mathbf{K}_1$, K_2 compose the computed gain $\mathbf{K}_{[1\times4]} = [\mathbf{K}_{1[1\times3]}\ K_2]$ of the augmented system.

Whereas the second term in (7) tends to drive the system to the reference, the first term represents the regulator that drives the states to zero. So, it is important to include the gain as in (5), where $\bar{N}$ must be now computed with $\mathbf{K}_1$.

Other approach evaluated to add an integral action is based on Mete and Gündes (2004). It consists in adding a PID to an initially designed stabilizing controller which doesn't have integral action. This is useful due to it is possible to apply an anti-windup action without affecting the closed-loop stability.

4.3 Feedforward gain of disturbance

Once the identified model also includes the collective pitch disturbance dynamics, it is reasonable trying to compensate the effect of disturbances in the output by computing a control action that counteracts this effect.

In order to compensate the steady-state effect of a constant disturbance, the necessary gain in control input which neutralizes the steady-state error for a given disturbance value is computed. This gain can be achieved by ratio of the DC gains of the transfer functions of each model input. For the system described by (2), this gain is given by:

$$u_{ff}(k) = -\frac{\mathbf{C}(\mathbf{I}-\mathbf{F})^{-1}\mathbf{G_{mr}}}{\mathbf{C}(\mathbf{I}-\mathbf{F})^{-1}\mathbf{G_{tr}}} u_{mr}(k). \qquad (8)$$

Therefore, the complete control law is given by:

$$\delta_{u_{tr}}(k) = -\mathbf{K}_1\mathbf{z}(k) - \frac{K_2}{q-1}(\mathbf{\Phi}\mathbf{z}(k) - r(k)) + \bar{N}r(k) + u_{ff}(k).$$

5 Experimental Results

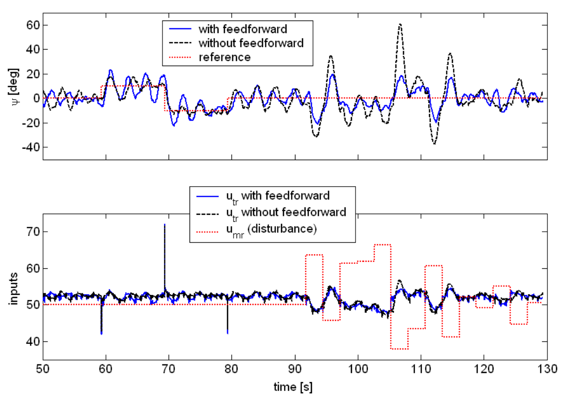


Figure 6: Comparison of the effect of the feedforward gain to disturbance rejection

Evaluation tests were carried out with four distinct yaw controllers: one PID tunned with the identified model, one LQG with integral action by state augmentation, one LQG with integral action by the two stages method, and the cascade controller used in the identification test. The yaw controllers were designed following the parameters below:

- PID : $K_p = 6$, $K_i = 1$ and $K_d = 2$.
- LQG with integral action by state augmentation :

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{R} = 50.$$

- LQG plus PID block : $\mathbf{Q} = 50 \cdot \mathbf{C}^T\mathbf{C}$, $\mathbf{R} = 1$, $K_p = 10$, $K_i = 6$ and $K_d = 0$.
- PI and PD in cascade:

  PI: $K_p = 2$, $K_i = 1$

  PD: $K_p = 8$, $K_d = 5.5$.

During experiments, the LQG rotor speed regulator was used to keep the rotor speed at 1510 *rpm*. This regulator is more detailed in Martins et al. (2006). All tests were made after driving the helicopter to its nominal operating condition, near hovering. These tests consisted in applying a step in reference position, and posteriorly a random sequence of values in the collective pitch input as disturbances. This sequence was the same for all tests.

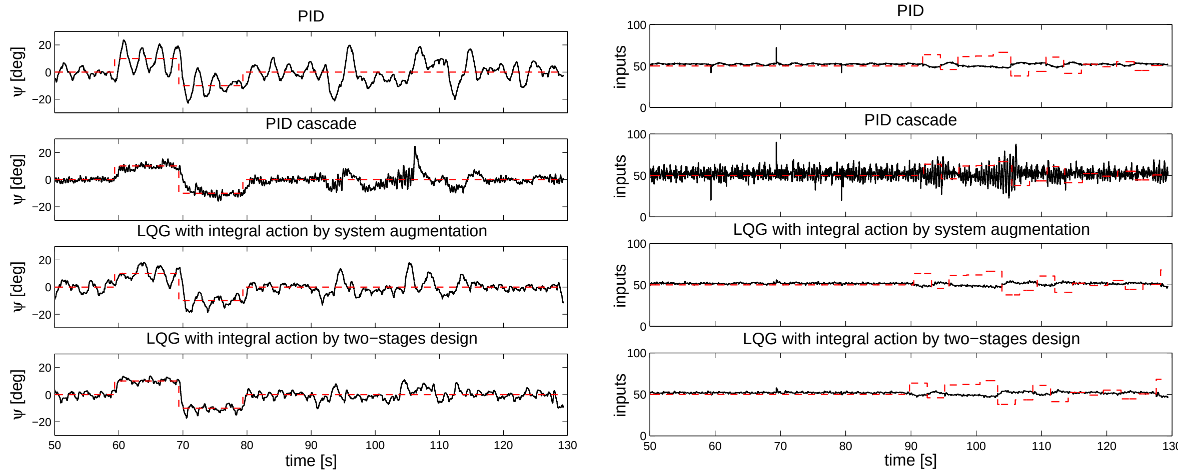
The evaluation of (8) gives a feedforward gain of $u_{ff}(k) = -0.1331 \cdot u_{mr}(k)$. Figure 6 shows the results for the PID controller with and without feedforward gain. As the results show a good improvement of disturbance rejection, this feedforward gain was used with all others yaw controllers.

In Figure 7(a), the experimental results for all yaw controllers are shown. As design requirements, it is desired a fast step response and small influence from collective pitch disturbance. Figure 7(b) shows the controllers input u_{tr} and the disturbance u_{mr}.

The results show a great performance of the LQG with PID block, once the oscillations were small using control input efforts much better when compared to the ones from PID in cascade. It is important to reduce servo damaging in long-term operations.

6 Conclusions

This work focused on the design of a yaw controller for small-scale helicopters. In this approach, a identification procedure for yaw dynamic model was proposed. The model compensates for collective pitch variation, which may be seen as an important disturbance, since the rotor



(a) (b)

Figure 7: The tests results. The yaw position are shown in (a): ψ (solid) and the reference (dashed). In (b) are the servos signals: u_{tr} (solid) and u_{mr} (dashed).

speed is maintained constant. Different control strategies were evaluated. The experimental results allowed comparison of the control strategies in respect to step response and robustness to disturbances.

This research is a part of an incremental approach for design of an aerial robot based on a helicopter model. Current activities focus on automatic control of the helicopter attitude.

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